Why Maxwell's Equations and the Lorentz Force Are not Fully Invariant under a Galilean Transformation

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When students first take their introductory courses in special relativity, they are told that in classical electromagnetism, Galilean transformations do not preserve equational relationships involving fields. This suggests that the subject is not Galilean invariant. Using Maxwell's theory, one can predict the electromagnetic waves, moving at a speed, c, in vacuo. In their simplest form, Maxwell's equations are only valid if they have a properly defined frame of reference, which is the luminiferous ether or a medium permitting the wave's propagation. Therefore, according to Galilean relativity, with reference to the ether, an observer not at rest has a different speed of motion than the electromagnetic waves. Following experimental tests, Michelson-Morley contracted Galilean's idea and introduced a new Lorentz invariance, which Einstein interpreted using new laws of physics. This introductory idea may lead students to think that classical electromagnetism's non-Galilean invariance entirely depends on the presence of the parameter, c, found in Maxwell's equations. To prevent this misinterpretation, it would be necessary to pedagogically invite students to view things differently. In particular, we can tell the students that the parameter, c, can be considered an attribute of Newtonian free space, and especially that its equation features two free space quantities: the vacuum permeability, μ0, and the vacuum permittivity, ε0.

$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ ……Equation 1.1

Since Newtonian physics considers a vacuum to be observer-invariant, both parameters can be regarded as being observer-invariant and can be determined separately. Based on this, Maxwell's parameter, c, is also observer-invariant and mimics a scalar invariant if frame transformation is the reference.

Since the above conclusion depends on the use of specific physical systems, it may seem contrived. However, the final result is independent of it. The scalar invariance of c also characterizes pre-relativistic physics, and the choice of SI units is useful for highlighting this fact. Once c's observer-independent attribute in pre-relativistic physics is recognized, students will acknowledge that the parameter's presence in Maxwell's equations does not depict classical electromagnetism's Galilean non-invariance. Obviously, the Lorentz invariance of Maxwell's theory is already known; but, how was Maxwell able to prove the Galilean non-invariance of classical electromagnetism, despite being ignorant of Lorentz transformations? This will be observed in the next segment.

There are three basic ingredients that pre-relativistic physicists use: 1) the Galilean transformation laws for Newtonian mechanics; 2) the classical electromagnetism equations postulated by Maxwell; and 3) electric charge invariance.

Galilean relativity

Under the set of transformations, t' = t+ a and x'= Rx – v_ot + b, where a ε R, R ε SO(3), and vo, bεR^3, Newtonian Mechanics is invariant. Since the transformations do not permit spatial contraction and time dilation, the spatial volume, V, is a Galilean invariant, given by V'=v.

Electric charge invariance

The conservation of electric charge under frame transformation is a principle of classical electromagnetism. Therefore, if V = V(t) is the spatial volume containing the Galilean invariant charge, q, and density, ϱ' = ϱ(t, x(t)). Let the charge current density be j= ϱv, then its Galilean transformation law would be: j= j-ϱv_o.

Maxwell's equations

$\nabla . B = 0$……………………..2.12

$\nabla \times E = -\frac{\partial B}{\partial t}$ …………..2.13

$\nabla \times E = -\frac{\varrho}{\varepsilon_o}$……………..2.14

$\nabla \times B = \mu_0 j + \frac{1 \partial E}{c^2 \partial t}$……2.15

Where B and E are the magnetic and electric fields, and C is Maxwell's constant.

Pre-relativistic requirement of Lorentz force invariance

Given that F = q(E+v+B) and F'=E' +v'xB' =E +vxB, B'(x', t') =B(x,t) and E'(x', t')=E(x,t) + v_oxB(x,t) are the solutions.

Other Galilean invariants include:

Magnetic Gauss' law; $\nabla' . B' = \nabla . B = 0$

Faraday's law; $\nabla \times E + \frac{\partial B}{\partial t} = 0$

Gauss law; $\varrho v_o = \varepsilon_o \{(v_o . \nabla)E + v_o \times [(\frac{\partial}{\partial t} + v_o . \nabla)B]\}$

Conclusion

Based on the information above, it is evident that a pre-relativistic physicist does not need knowledge about Lorentz transformations to prove that classical electromagnetism is Galilean non-invariant. Not surprisingly, it is easy to perform a pre-relativistic set of Galilean transformations and notice that Maxwell's second pair of equations fail to satisfy the condition of Galilean invariance.